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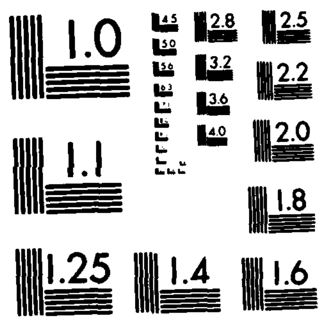
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During the period of research, efforts were made in the areas of (1) controllability of infinite dimensional bilinear control systems (2) singular perturbations approach to optimization problems with non-convex cost, (3) nonlinear continuum mechanics and phase transitions, (4) optimal control of a problem arising in robotics. Results were obtained in all areas using various methods of nonlinear analysis.

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INTERIM TECHNICAL PROGRESS REPORT

for the period ending 14 June 1983

AFOSR-81-0172

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Troy, N.Y. 12181



Title of Research:

Nonlinear Systems in Infinite Dimensional State Spaces

## Abstract

During the period of research, efforts were made in the areas of (1) controllability of infinite dimensional bilinear control systems (2) singular perturbations approach to optimization problems with non-convex cost, (3) nonlinear continuum mechanics and phase transitions, (4) optimal control of a problem arising in robotics. Results were obtained in all areas using various methods of nonlinear analysis.

1. Controllability of infinite dimensional bilinear systems

A standard equation governing small transverse deflections of a beam is

$$u_{tt} + u_{xxxx} + p(t)u_{xx} = 0 \qquad 0 < x < 1 \tag{1.1}$$

where $u(x,t)$ denotes the deflection, $p(t)$ represents the axial load (a real valued function). A natural control problem one might consider is the following: Given initial data

$$u(x,0) = u_0(x) , \quad u_t(x,0) = u_1(x) \tag{1.2}$$

consistent with some reasonable set of boundary conditions and terminal data

$$u(x,T) = u_T(x) , \quad u_t(x,T) = u'_T(x) \tag{1.3}$$

find a control $p(t) \in L^2[0,T]$ which accomplishes this transfer.

For example, if we consider the cased of hinged boundaries $u = u_{xx} = 0$ at $x = 0, 1$ and expand

$$u(x,t) = \sum_{u=1}^{\infty} u_n(t)\phi_n(x) \tag{1.4}$$

where $\phi_n(x) = \sqrt{2} \sin n\pi x$. We see the $\{u_n(t)\}$ satisfies the infinite system of ordinary differential equations

$$\ddot{u}_n(t) + n^4\pi^4 u_n(t) - p(t)n^2\pi^2 u_n(t) = 0 , \tag{1.5}$$

$n = 1,2,3,\ldots$ . Now our goal of state transfer can be viewed as desiring one control $p(t)$ which transfers each Fourier coefficient of the initial data to the desired Fourier coefficient of the terminal data. This problem is far from trivial since (i) we are attempting to control an <u>infinite</u> number of

modes with one control $p(t)$, (ii) the input-output relation $p \to \{u_n\}$ is nonlinear.

For problems of the type where the modes decouple as in (1.5), Ball, Marsden, and Slemrod [1] developed a theory which identified attainable terminal data for a reasonable class of controls. In research done this past year, Slemrod [2] has extended these results to a class of control systems for which the modes do not decouple. For example, Slemrod theory allows identification of attainable terminal data for control systems of the form

$$\ddot{u}_n(t) + \lambda_n^2 u_n(t) + p(t) \sum_{m=1}^{\infty} B_{mn} \lambda_m u_m(t) = 0 \tag{1.6}$$

where the coupling coefficients $B_{mn}$ satisfy special hypotheses. Slemrod applied his results to a class of wave equations with mixed boundary conditions. The main technique was to perform a change of variables that allowed application of the classical Fredholm-Riesz-Schauder theory for a linearized problem near $p \equiv 0$ and then application of "local onto" theorem for obtaining the nonlinear result..

Future research will be based on this approach, possibly employing the Leray-Schauder degree theory.

2. Singular perturbations approach to optimization problems.

In this research Slemrod (in collaboration with J. Carr of Heriot-Watt University, Edinburgh, U.K. and M. E. Gurtin, Carnegie-Mellon Univ., Pittsburgh, Pa.), we are considering the following problem motivated by the equilibrium theory of phase transitions.

Find a function $w(x)$, $u < x < 1$, so that $w(x)$ minimizes the "cost" or "energy" functional

$$E(w) = \int_0^1 W(w'(x)dx$$

with prescribed boundary conditions. If W is convex, a minimum always exists. However, if W is non-convex, typically only a "relaxed" solution exists, i.e., a solution to the problem exists for W replaced by its convex hull. In our research we have studied the singularly perturbed problem of minimizing

$$\int_0^1 \varepsilon w''^2_\varepsilon + W(w'_\varepsilon(x)dx \quad , \quad \varepsilon > 0$$

subject to boundary conditions. We have completely analyzed the effect of such higher gradients on the solutions for small $\varepsilon > 0$. In fact, we have shown that introduction of the $\varepsilon$ term appreciably changes the situation in that solutions to the second problem are essentially unique and have unique limits as $\varepsilon \to 0$. In this sense, incorporating the physically expected $\varepsilon$ term in the cost and <u>then</u> passing to the limit produces an essentially unique solution whereas studying the relaxed problem immediately produces an infinity of solutions. We view our approach as a natural "admissibility" criteria for non-convex cost problems.

One paper has already been completed [3]. A second is in preparation.

3. Nonlinear continuum mechanics and phase transitions.

In the study of phase transitions in fluids a typical model given in classical thermodynamics is provided by the equilibrium configurations of a van der Waals fluid. Such equilibria show the fluid may exist in two phases,

liquid and vapor. Relatively untouched in classical thermodynamics is the non-equilibrium case, i.e., how the dynamics of phase transitions proceed. The goal in this research was to study precisely this subject from continuum mechanics (not statistical mechanics) point of view.

While earlier work has dealt with the "structure" of interphase shocks, Slemrod has lately been considering the role of viscosity and capillarity effects on the full hydrodynamic equations of mass, momentum, entropy, and energy. Specifically, he has been considering the problems of (i) whether inviscid hydrodynamics is really a good model for nonequilibrium phase transitions and (ii) how should we compute numerically solutions to the hydrodynamic equations. Tentative answers to these problems seem to be (i) inviscid hydrodynamics is <u>not</u> a good model for nonequilibrium phase transitions since omitted viscous-capillarity terms are crucial to the limiting results and (ii) a naive Lax-Friedrichs scheme may be a reasonable way of performing isothermal computations.

In related work with J. K. Hunter [4], Slemrod has considered dynamic phase transitions in polymers. Specifically, they have proposed a phenomological explanation for the appearance of the "ripple" phenomena in melt fracture of certain polymers. Their main idea was to consider a nonlinear monotone rate of strain history functional for the shearing stress in an isotropic viscoelastic fluid. This led them to consider the initial history valued problem

$$v_t(x,t) = \frac{\partial}{\partial x} \int_0^\infty e^{-\lambda s} \, \sigma(v_x(x,t-s))ds + \gamma \qquad (3.1)$$

$$v(x,t) = v_0(x,t) \, , \quad -\infty < t < 0 \quad ,$$

where $v(x,t)$ is the fluid velocity at a point $x$ at time $t$, $\sigma$ is a non-monotone function, $\gamma$ is the applied pressure gradient. Slemrod and Hunter showed that solutions of (3.1) exhibit hysteresis and phase separation in a manner similar to observed experimental results.

4. Optimal control problems arising in robotics.

Slemrod supervised a Master in Applied Mathematics project of Gabriel Amazigo on modeling and control of a deformable arm. In this project Amazigo first used standard Timoshenko beam theory to obtain a set of ordinary differential equations governing the lowest modes of an axially and flexurally deformable arm. In the second part of the project Amazigo used the Pontrayagin maximum principle to find an optimal control to bring the arm from a deformed state back to its state of rest. This work is currently being written up by Amazigo.

## References


1. Ball, J. M., J. E. Marsden and M. Slemrod, Controllability for distributed bilinear systems, SIAM J. Control and Optimization, 1982, 20, 575-597.

2. Slemrod, M., Controllability for a class of nondiagonal hyperbolic distributed bilinear systems, in preparation.

3. Carr, J., M. E. Gurtin and M. Slemrod, One dimensional structured phase transformations under prescribed loads, submitted to J. of Elasticity.

4. Hunter, J. K., M. Slemrod, Visco-elastic fluid flow exhibiting hysteretic phase changes, to appear Physics of Fluids, August, 1983.


## Summary List of Publications

Publications [2,3,4] above were completed during 7/82-7/83 under AFOSR-81-0172.

## Interactions

M. Slemrod spent 1/4 of the academic year as a senior fellow at the Institute for Mathematics and Its Applications, Univ. of Minnesota, Minneapolis, Minn. He also gave the following invited addreses:

IEEE Control and Decision Conference, Dec. 15, 1982, Orlando, Fla., Stabilization of Nonlinear Wave Equations.

Control Theory Seminar, Univ. of Minn., Dept. of Electrical Eng., Control and Stability of Nonlinear Wave Equations, March 22, 1982.

Conference on Physical Partial Differential Equations, Univ. of W. Va., Morganstown, W. Va., July 7, 1983, Dynamics of Phase Transitions.